Water and wastewater treatment – Homework 7 - Solutions

1. Analysis of particle size

The table below shows information on particle size distribution in a raw water and a flocculated/filtered water

Diameter (μm)	Cumulative number of particles in raw water (#/mL)	Cumulative number in flocculated/filtered water (#/mL)
< 5	55889	365
< 7.5	89943	544
< 15	109321	689
< 30	111581	723
< 50	111756	728
< 70	111777	729

a) Plot the particle distribution (log particle size distribution function vs log particle diameter). Hint: Make a table with the critical parameters Δd_p, ΔN, $\log(\Delta N/\Delta d_p)$ and log(mean d_p) of the raw water and the filtered water.

Raw water

d_p μm	Δd_p	ΔN	$\log(\Delta N/\Delta d_p)$	log(mean d_p)*
5				
7.5	2.5	34054	4.1342	0.787
15	7.5	19378	3.4122	1.026
30	15	2260	2.1780	1.327
50	20	175	0.942	1.588
70	20	21	0.021	1.772

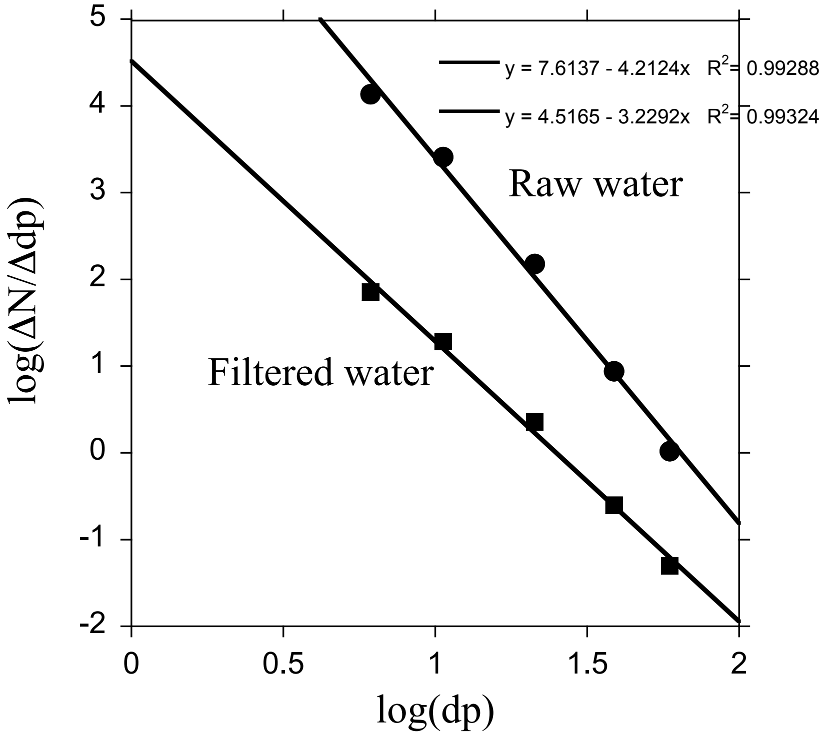
*example: mean d_p (5-7.5 μm) = $\sqrt{5\times7.5}$ (geometric mean)

Filtered water

d_p μm	Δdp	ΔN	$\log(\Delta N/\Delta dp)$	$\log(d_p')$*
5				
7.5	2.5	179	1.855	0.787
15	7.5	145	1.286	1.026
30	15	34	0.355	1.327
50	20	5	-0.602	1.588
70	20	1	-1.301	1.772

*example: d_p' (5-7.5 μm) = $\sqrt{5\times7.5}$ (geometric mean)

b) Determine the parameters A and β. How do you interpret these values?



Raw water: logA=7.6, β= 4.2
Filtered water: logA=4.5, β= 3.2
A: With increasing A, the total number of particles increases
β <1 large particles dominate
β =1 all particles sizes are equally represented
β >1 smaller particles dominate
From raw water to filtered water the particle number diminishes dramatically. The particle size distribution is shifted towards larger particles (β: 4.2 -> 3.2)

2. Sedimentation
a) Calculate the terminal settling velocity for a particle in water at 20ºC, using Stoke's law. The particle has a diameter of 25 μm and a density of 2000 kg/m³. The density of water is 1000 kg/m³ and at 20ºC, μ is 1.002 x 10^{-3} Ns/m².

According to Stoke's law:

$$v_s = \frac{g(\rho_p - \rho_w)d_p^2}{18\mu} = \frac{9.81(2000-1000)(2.5\times10^{-5})^2}{18(1.002\times10^{-3})} = 3.4\times10^{-4}\,\mathrm{m/s} \;= 1.22\ \mathrm{m/h}$$

b) Is the assumption of the Stoke's law correct (laminar conditions)?

Calculation of the Reynolds number:

$$\text{Re} = \frac{\rho_w d_p v_s}{\mu} = \frac{1000 \times (2.5 \times 10^{-5}) \times (3.4 \times 10^{-4})}{1.002 \times 10^{-3}} = 8.5 \times 10^{-3}$$

The Reynolds number is <1 and therefore laminar flow exists and the Stoke's law can be applied.

c) What are the consequences for the terminal settling velocity if the temperature changes from 20ºC to 5ºC? Assumptions: Density of water remains constant, μ at 5ºC is 1.52×10^{-3} Ns/m^2.

$$\frac{v_{s,5}}{v_{s,20}} = \frac{\mu_{20}}{\mu_5} \Rightarrow v_{s,5} = \frac{\mu_{20}}{\mu_5} v_{s,20} = 0.66 \times 3.4 \times 10^{-4} \text{m/s} = 2.24 \times 10^{-4} \text{m/s}$$
$$= 0.8 \text{ m/h}$$

The terminal settling velocity at 5ºC is only about 70% of that at 20ºC.

d) Calculate the same parameters for a particle diameter of 250 μm at 20°C. Check again if the assumptions for the Stoke's law are fulfilled.

According to the Stoke's equation, $v_s = 0.034$ m/s
Re can be calculated according to the above equation: Re = 8.48
Since Re is >1, C_D has to be calculated by:

$$C_D = \frac{24}{\text{Re}} + \frac{3}{\sqrt{\text{Re}}} + 0.34 = 4.2$$

v_s has to be calculated by the Newton equation

$$v_s = \sqrt{\frac{4g(\rho_p - \rho_w)d_p}{3C_D\rho_w}}$$
= 0.028 m/s
Based on this value the Re can be calculated: Re = 6.99
C_D then becomes: $C_D = 4.9$
$v_s = 0.026$ m/s
This iteration is continued until v_s converges. The results are shown in the Table below.

Trial	Re	C_D	v_s m/s
0	8.48	4.2	0.028
1	6.99	4.9	0.026
2	6.49	5.2	0.025
3	6.24	5.4	0.0246
4	6.14	5.46	0.0245

The settling velocity of the 250 μm particle is about 0.0245 m/s = 88.2 m/h

e) What is the minimum length (depth 4 m, width 10m) of a settling tank for the removal of the 25 μm particles at 20°C and 5°C? Flow rate of water 500 m^3/h.

$$v_c = \frac{Q}{A} = \frac{Q}{l \times w} = v_s$$

$$l = \frac{Q}{v_s \times w}$$

$$l_{20} = \frac{500}{3.4 \times 10^{-4} \times 3600 \times 10} = 41 \text{ m}$$

$$l_5 = \frac{500}{2.24 \times 10^{-4} \times 3600 \times 10} = 62 \text{ m}$$

f) Calculate the particle removal in the sedimentation basin from e) (depth 4 m, width 10 m, length 40 m, water flow rate 500 m^3/h, T= 20°C). The particle concentration and size is given in the Table below. The particle density is 2000 kg/m^3.

Diameter μm	Number of particles 1/L
10	$1x10^6$
20	$3x10^6$
25	$2x10^6$
30	$4x10^6$
50	$2x10^5$
100	$1x10^6$

i) Calculate the critical settling velocity

$$v_c = \frac{Q}{A} = 1.25 \text{ m/h}$$

ii) Calculate the settling velocities of the particles and the fraction of the particles removed. What is the number of remaining particles after the sedimentation tank?

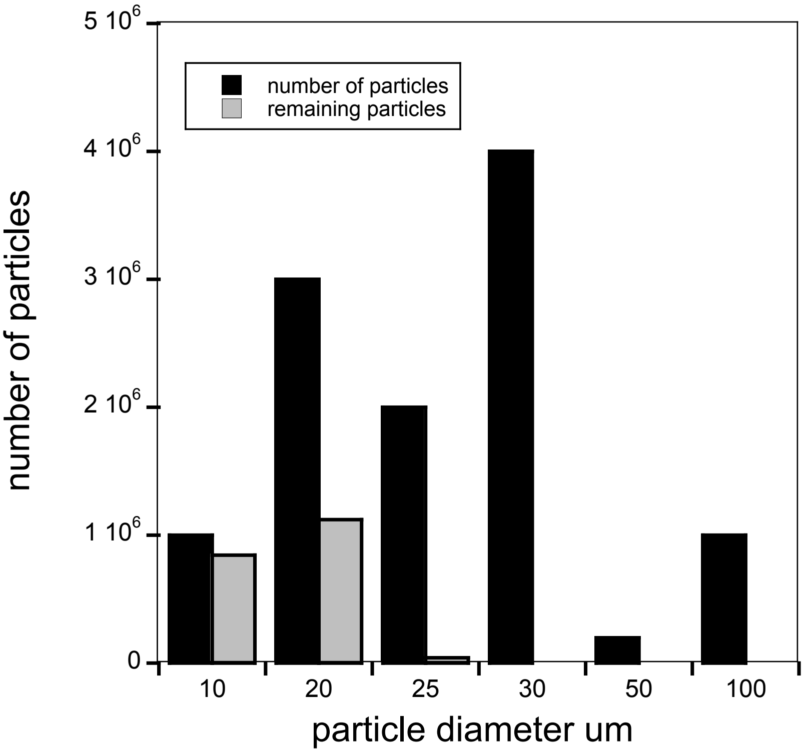
The fraction of the particles removed in the sedimentation tank is given by:

$$f = \frac{v_s}{v_c}$$

Diameter μm	Settling velocity m/h	Fraction of particles removed	Remaining particles after sedimentation 1/L
10	0.196	0.16	8.43e+05
20	0.783	0.63	1.12e+06
25	1.22	0.98	4.19e+04
30	1.76	1.0	0.00e+00
50	4.90	1.0	0.00e+00
100	19.6	1.0	0.00e+00

iii) Plot the number of particles as a function of the particle diameter before and after the sedimentation tank.

A graph of the particle distribution before and after the sedimentation tank is shown below.



iv) What is the overall particle removal in percent?
Percent particle removal:
Number of particles in influent to sedimentation: 1.12×10^7 /L
Number of particles after sedimentation: 2×10^6 /L
This results in an overall removal of 82%.

3. Filtration

A deep bed rapid filter consists of 1 m sand with an average diameter of the sand grains of 0.5 mm. The filtration rate is 10m/h.

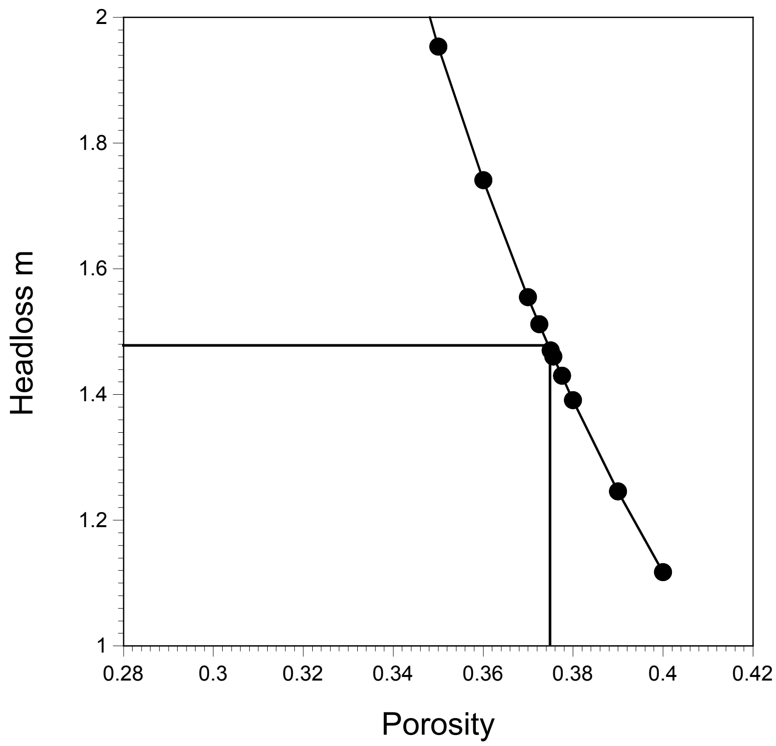
a) Calculate the clean-bed head loss for a temperature of 5ºC. Use average values for κ_v, κ_I and ε and value for the dynamic viscosity from the lecture notes.

$$\text{head loss } h_L = \kappa_V \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{\mu L v}{\rho_w g d^2} + \kappa_I \frac{1-\varepsilon}{\varepsilon^3} \frac{L v^2}{g d}$$

$$= \frac{112 \times (0.6)^2}{0.4^3} \times \frac{0.00155 \times 1 \times (10/3600)}{1000 \times 9.81 \times (0.5 \times 10^{-3})^2} + \frac{2.25 \times 0.6 \times 1 \times (10/3600)^2}{(0.4)^3 \times 9.81 \times 0.5 \times 10^{-3}} = 1.14 \text{ m}$$

b) Plot the headloss as a function of porosity (using the parameters above). For the headloss to be 30% higher than in question a), how much would the porosity have to decrease (assuming that all other parameters remain constant)?

The graph below shows the head loss as a function of the porosity ε. A 30% increase in head loss (1.46 m) is reached for a relatively small change in porosity from 0.4 to 0.376.



c) Calculate the head loss for a non-homogeneous deposition of particles (the filter influent water contains 5×10^5 particles/L with a diameter of 50 µm and a density of 2000 kg/m^3). The table below shows the relative removal in each layer of the filter after 2h. Use the calculated cumulative amount of material to estimate the incremental head loss from the figure below.

Filtration depth cm	C_x/C_o
0	1
5	0.85
10	0.78
15	0.72
20	0.68
30	0.65
40	0.62
60	0.62
80	0.62
100	0.62

Volume of individual particles: V=1/6 x π x d^3 = 1/6 x π x $(5x10^{-5})^3$ = 6.5 x 10^{-14} m^3
Particle mass: 6.5 x 10^{-14} m^3 x 2000 kg/m^3 = 1.3 x 10^{-10} kg = 1.3 x 10^{-7} g
1L of water contains 5 x 10^5 particles: 65 mg/L TS

Calculation of the accumulation of mass in filter (mg/cm^3):

$$\Delta q = v\frac{\Delta C}{\Delta x}\Delta t$$

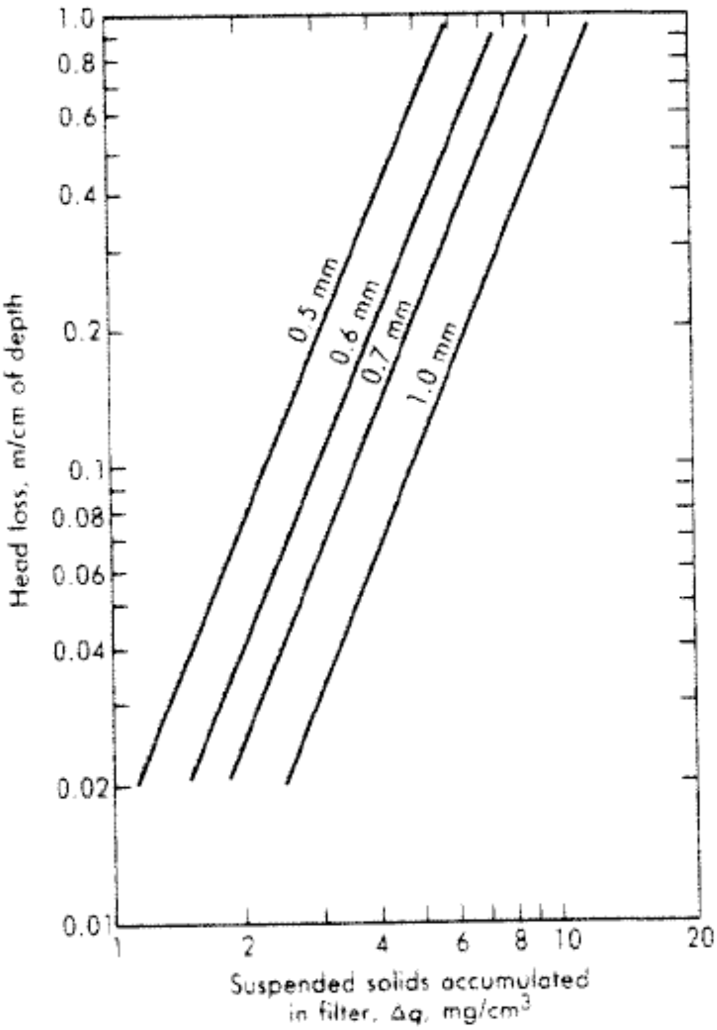
$$v = 10\text{m/h} = 10\text{m}^3/\text{m}^2\cdot\text{h} = 1\text{L/cm}^2\cdot\text{h}$$

zone 0-5 cm:

$$\Delta q = 1\cdot\frac{0.15\cdot 65}{5}\cdot 2 = 3.9\ \text{mg/cm}^3$$

From the figure below, an accumulation of 3.9 mg/cm^3 suspended solid yields to a head loss of 0.4 m/cm. Therefore, the head loss between 0-5 cm is 5 x 0.4m = 2 m
The head loss calculations for the other depths in the filter are shown in Table below.

Filtration zone cm	Head loss m
0 -5	2
5-10	0.05 x 5 = 0.25
10-15	0.04 x 5 = 0.2
15-20	0.02 x 5 = 0.1
20-30	-
30-40	-
40-60	-
60-80	-
80-100	-
Total head loss 0-100	2.55 m



Head loss, m/cm of depth
1.0
0.8
0.6
0.4
0.2
0.1
0.08
0.06
0.04
0.02
0.01
0.5 mm
0.6 mm
0.7 mm
1.0 mm
1
2
4
6
8
10
20
Suspended solids accumulated
in filter, Δq, mg/cm3